Improved Flow Curve Determination Using the Bulge Test Combined with Optical Measurement Systems and Compensation Strategies

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Abstract. Commonly, a tensile test is performed to determine a flow curve for modeling the plastic hardening effect of sheet metals. However, only as long as the specimen is uniformly elongated one can assume an uniaxial stress state. Due to this limitation the maximum equivalent plastic strain obtained from standard tensile tests is typically lower than the one, which appears in industrial deep drawing processes. The bulge test offers experimental data for modeling the plastic hardening behavior at more elevated strain levels. The flow curve obtained from bulge tests, is usually derived by applying the membrane theory. Nowadays, optical measuring systems like ARAMIS [1] are available and utilized for the determination of the time dependent deformation field. The exploitation of the measured data enables the computation of the associated thickness and the curvature at the top of the dome with respect to the measured surface. The pressure, induced by the testing machine, acts at the inner surface. According to the membrane theory, these quantities have to refer to the middle layer. In this paper different methods for compensating this contradiction are discussed. As opposed to the first experimental comparison in [2] the validation of the different approaches is carried out by a simulation based procedure.

Introduction

The die and the binder of the bulge test are typically rotational symmetric and the specimen is formed by an increasing level of oil pressure (Fig. 1). This experimental setup induces a biaxial stress state at the top of dome, which is not affected by friction. During the forming process, the deformation field in the region of the top of the dome and the oil pressure are recorded. All results of the optical measurement are derived from the outer surface and are typically directly used for the calculation of the flow curve according to the upcoming ISO 16808. For the determination of the flow curve based on the bulge test an analytical approach is necessary in order to compute the stress state at the top of the dome. If the ratio between the sheet metal thickness and the bulge diameter is small, bending stresses can be neglected [4] and consequently the requirements for deploying the membrane theory for the determination of the biaxial stress state are fulfilled:

\[ \sigma_B = \frac{p \rho}{2t} \]  

Eq. 1 is valid, if both principal stresses are assumed to be equal (\( \sigma_B = \sigma_1 = \sigma_2 \)) [5]. Furthermore the application of Eq. 1 implies the assumption that the shape of the dome in the pole zone is spherical. According to the membrane theory the radius \( \rho \) refers to the middle layer of the material and also the pressure acts on this layer. The thickness can be calculated by assuming plastic incompressibility using the measured major and minor strains (\( \varepsilon_1, \varepsilon_2 \))
\[ I = I_0 e^{\varepsilon_1}, \quad \varepsilon_3 = -(\varepsilon_1 + \varepsilon_2). \] (2)

In order to investigate the applicability of the membrane theory for the computation of the biaxial stress state Volk et al. [4] suggested a simulation based validation procedure. Thereby a forming simulation of a bulge test is performed. The geometry of the specimen, which is represented by a finite element mesh, is exported at given points in time during the forming process. For each of these exported meshes the nodal coordinates are extracted. The time dependent positions of these points describe the deformation of specimen during the forming operation. The resulting point sets can be processed by the same procedure as for the computation of the flow curve based on optically measured data. If an isotropic material model is deployed, the algorithm for the determination of the flow curve should be able to reproduce the flow curve defined in the material card based on the exported point sets and the pressure progression. In [4] shell elements are applied, whose reference surface is coincident with the middle layer. As a consequence the mentioned requirements of Eq. 1 are complied. According to the investigations of Volk et al. [4] the membrane theory is applicable for the sheet metal thicknesses of 0.5 mm, 1.0 mm and 3.0 mm (d_{\text{die}} = 200 mm).

![Fig. 1: Principle and experimental setup of the laboratory of BMW using the optical measuring system ARAMIS](image)

**Correction proposals for the flow curve calculation**

As mentioned, the optical measurement system in the laboratory gives the time dependent deformation field on the visible surface (outer surface) and the pressure acts on the opposite side (inner surface). Consequently, the measured quantities do not refer to the middle layer, as required for the application of Eq. 1. If this contradiction is neglected, a negative effect on the quality of the computed biaxial stress state is expected. In this context the question arises, whether the measured quantities can be corrected in order to relate them with the middle layer. In [2,7] different approaches for correcting the measured data are discussed.

**Influence of bending (=material thickness) on the determined strain and material thinning. Approach – S.**

If an ARAMIS system is used for the optical measurement of the deformation field, a function for “material thickness compensation” can be activated. A non-trivial algorithm in ARAMIS is able to shift the measured surface coordinates to the values on the middle layer \( m \) of the blank. In this case the strain field is computed according to the deformation field referring to the middle layer. This eliminates the bending influence in the strain field based on the local curvature. This new strain field
(\varepsilon_{1m}, \varepsilon_{2m}) \) defines a bending compensated thickness strain field and consequently an improved computation of the material thickness according to Eq. 2.

**Approach - S*. A simplified approach for compensating the bending strain effect is given in [7]:

\[ t_m = t_0 e^{\varepsilon_m}, \quad \varepsilon_{3m} = \varepsilon_3 - 2 \ln \left(1 - \frac{t_0}{2\rho} e^{\varepsilon_3}\right), \quad \varepsilon_3 = -(\varepsilon_1 + \varepsilon_2). \] (3)

**Influence of thickness for curvature determination. Approach – R.**

Using ARAMIS with material thickness compensation, the determination of the curvature can be directly realized with respect to the middle layer coordinates.

**Approach – R*. In [2,7] a second, simplified approach for an optimized curvature is given by:

\[ \rho_m^* = \rho_s - \frac{t_m}{2}. \] (4)

A significant deviation between these two approaches is observed in [2]. The approach R was evaluated as more reliable. Due to the local thinning at the apex of the dome, the simple approach does not give reasonable results for the real radius of the middle layer \( \rho_m \) for higher strain values. The principle difference between the real \( \rho_m \) in the case of a localized thinning and \( \rho_m^* \) calculated without localized thinning is shown in Fig. 2. There the radius \( \rho_s \) of the outer layer is printed with a constant value. The simple approach can only be used in the case of a homogeneous thinning of the material (left side). On the right side the case of a localized thinning at the apex of the dome is shown and \( \rho_m \) is much smaller than \( \rho_m^* \) from the left side.

![Diagram showing homogeneous and localized thinning](image)

Fig. 2: Real \( \rho_m \) for the case of localized thinning defined by middle layer coordinates (right side) in comparison to simple approach by Eq. 4, which is only valid for homogenous thinning (left side) [2]

**Thickness based compensation for pressure. Approach – P.**

Based on the idea that the pressure has to act on the inner layer of the material in [2,7] a simple compensation of the material thickness for the pressure was proposed from the general balance of forces for thin shells (see Eq. 1). Eq. 5 is valid for the combination of the correction approaches R and S. For other compensation strategies a suitable pressure compensation has to be derived

\[ \sigma = \frac{P \cdot \rho_m}{2t_m} \left(1 - \frac{t_m}{2\rho_m}\right)^2. \] (5)
Elastic strain compensation. Approach – E.

If the results have to present only the plastic components of the deformation, a compensation of the elastic strains is proposed in [7]. With this approach a flow curve including elastic strains can be transferred to a flow curve without elastic strains by using the Hooke's law comprising the model parameters Young's modulus and Poisson's ratio.

Procedure for investigating compensation strategies

For validating the improvement of the different approaches for compensating the measured quantities the real flow curve of a material would be helpful. However, such data is not available. Alternatively for the assessment of the mentioned correction methods the same simulation based investigation can be performed as shown above [4]. However, the shell elements have to be exchanged by volume elements (Fig. 3), which enable to extract the corresponding nodal displacements on the outer surface of the specimen and allow defining a pressure boundary condition acting on the inner surface. As a consequence, the simulation based investigation reflects the conditions of the real experiment. As large differences regarding the edge lengths of the volume elements lead to a reduction of the accuracy of the numerical integration [6], the initial blank comprises cubic shaped volume elements. For modeling the material response of the specimen a hypoelasto-plastic material model is deployed, implying a von Mises yield locus and an isotropic hardening law. The significance of the mentioned contradiction regarding the application of Eq. 1 by directly using the measured quantities is expected to increase with the material thickness. It seems to be reasonable to perform the investigations of the correction approaches on the basis of the upper limit of the material thickness according to the upcoming ISO 16808, which is given by \( t_0 = d_{die}/33 \) mm. This study is aligned to the experimental setup of the laboratory of BMW (Fig. 1) comprising a die diameter of \( d_{die} = 200\text{mm} \). As a consequence, the application of \( t_{max} = d_{die}/33 \) leads to a maximum sheet thickness of \( t_{max} = 6\text{mm} \). The subsequently presented results are based on this sheet thickness and the investigated flow curve is derived from the interstitial free mild steel DX54. Finally the evaluation of the introduced compensation strategies can be performed by comparing the computed flow curve, resulting from the time dependent nodal coordinates and the pressure progression of the simulation, with the flow curve defined in the material card.

![Fig. 3: Forming simulation of bulge test using volume elements](image)

Validation of the applicability of the membrane theory

As the evaluation of the applicability of the membrane theory in [4] is limited to a maximum sheet thickness of 3 mm, subsequently this question is studied again on the basis of a sheet thickness of 6 mm. However, in this paper volume elements are applied, which also enable to export the deformation field of corresponding points on the middle layer. Additionally, this element type enables to define the pressure boundary condition according to the membrane theory. As a consequence, the simulation model based on volume elements complies all the requirements of the membrane theory. In Fig. 4 a comparison between the flow curve of the material card “Ref”
(reference curve) and the recalculated flow curve “ML” is shown (ML: middle layer). “ML-E” is a second recalculated flow curve, which is complemented by the elimination of the elastic strains based on approach E.

Fig. 4: Comparison of Ref: flow curve from the material card, ML: recalculated flow curve based on Eq. 1 with respect to the middle layer and ML-E recalculated flow curve without the elastic strains

In the left graph the stress values of the three curves match well. In the right graph the relative deviations between the recalculated curves and the reference curve are shown. The small deviation of the recalculated curve (maximum deviation: 2%) indicates the validity of the membrane theory for the determination of the biaxial stress state at the top of the dome. If the elastic part of the strain state is additionally eliminated (curve “ML-E” in Fig. 4) the deviations are reduced regarding the whole curve and the maximum deviation is approx. 1% (in the zone of low strain levels). As the flow curve of the material card is defined without the elastic part of the equivalent strains a compensation of the elastic part is expected to be reasonable. This is confirmed by the curves “Ref” and “ML-E” in Fig. 4.

Validation of the different correction approaches concerning the pressure. Approach – P

For the validation of approach P (thickness based compensation for pressure) a second similar simulation with volume elements is performed. The pressure boundary condition is defined on the inner surface. For both simulations the deformation field of the corresponding points on the middle layer are used to recalculate the flow curve and are shown in Fig. 5.
Fig. 5: Comparison of recalculated flow curves for pressure acting on the inner surface and for pressure acting on the middle layer.

In the left graph the differences between the two curves are not visible. According to the right graph the relative difference is negligible. Based on this numerical results it can be assumed, that an additional compensation for the pressure acting surface as suggested in [2,7] should not be helpful. Thus it cannot be expected that a compensation for the pressure by approach P can increase the quality of a flow curve in the case of thick materials.

**Validation of the different correction approaches Approaches S, S*, R, R***

For the validation of the other approaches the same simulation with the volume elements is taken. In this case the deformation fields of corresponding points on the outer surface (OS) are used to calculate a flow curve similar to the post processing of a real measurement. The recalculated flow curve OS is shown in Fig. 6. As a reference for the success of the different approaches the flow curve from Fig. 4 (ML) is also printed as target of the correction. By adding different approaches (S, S*, R, R*) to the calculation of OS modified flow curves are obtained and shown in Fig. 6.

Fig. 6: Comparison of recalculated flow curves: ML reference curve based on middle layer data, OS outer surface data with different added approaches.

Approach S and S* deliver comparable results for the apex of the dome (no difference visible in Fig. 6 (right graph). A combination of S and R (flow curve OS-S-R) is very close to ML and seems to be the optimal correction. An additional correction of the pressure is not needed. Using the ARAMIS thickness compensation S and R are typically performed together. The correction R* underestimates the influence of local thinning for the whole curve. If the curves should present only the plastic components of the deformation, for all curves in Fig. 6 an additional use of approach E is necessary. E.g. this will correct OS-S-R of Fig. 6 similar as ML towards Ref (see Fig. 4).
Fig. 7: Comparison of recalculated flow curves: ML reference curve based on middle layer data, OS outer surface data with different added approaches. The right graph shows the differences between the OS curves and the ML curve.

**Approach P in combination with S, S*, R, R*.**

By adding approach P to the calculation of OS with (S, S*, R, R*) some more modified flow curves are calculated and shown in Fig. 7. As assumed before the additional use of approach P does not result in any positive effect for the correction of the flow curves.

**Summary**

In this study, the applicability of the membrane theory can be confirmed for the maximum permissible sheet thickness according to the upcoming ISO 16808 regarding a die diameter $d_{die} = 200mm$. Furthermore, it is shown on the basis of a forming simulation that the contradiction between the experimental setup and the requirements of the membrane theory can affect the quality of the determined flow curve. The authors recommend computing the curvature at the top of the dome with respect to the middle layer in consideration of local thinning. Additionally, the computation of the thickness should be performed on the basis of the strain field, which refers to the middle layer. The compensation of the pressure does not seem to be necessary.

**References**


